

**Amendments to the Specification:**

On page 2, please replace the paragraph beginning "One proposed application" (line 16) with the following paragraph:

One proposed application of a quantum computer is the factoring of large numbers. In such an application, a quantum computer could render obsolete all existing encryption schemes that use the "public key" method. In another application, quantum computers (or even a smaller scale device such as a quantum repeater) could enable absolutely safe communication channels where a message, in principle, cannot be intercepted without being destroyed in the process. *See, e.g., H.J. Briegel et al., LANL preprint quant-ph/9803056 (1998) and the references therein.*

On page 7, lines 16, please replace the paragraph beginning "A phase shifter" with the following paragraph:

Some embodiments of qubits according to the present invention include an s-wave (for example, niobium, aluminum, lead, mercury, or tin) superconducting structure that includes an asymmetric four-terminal junction with all terminals connected by constriction junctions. Two of the terminals can be joined to form a superconducting loop and the other two terminals can be coupled to a source of transport current. The superconducting loop includes a phase shifter, which may consist of a S-N-D-N-S (for example, niobium / gold / ~~Y-Ba<sub>2</sub>Cu<sub>3</sub>O<sub>7-x</sub>~~ YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7-x</sub> / gold / niobium) junction. If the incoming current is parallel to the a (or b) crystallographic direction of the d-wave material, and the outgoing current is parallel to the b (or a) crystallographic direction of the d-wave material, this S-N-D-N-S junction can give a phase shift of  $\pi$ . Choosing the incoming and outgoing currents to be at any arbitrary angle to each other in the a-b plane in this embodiment allows a more general phase shift.

On page 11, please insert the following paragraph immediately after line 6 and before line 7:

FIG. 14. 4-terminal Josephson junctions. (a) Junction with microbridges. (b) Mesoscopic junction with two-dimensional electron gas (2DEG) and symmetric current configuration. In this configuration, coupling between currents  $I$  and  $J$  inside the normal region is local ( $\delta=0$ ). (c) Mesoscopic junction in the asymmetric currents configuration. Coupling is non-local ( $\delta\neq 0$ ).

On page 11, please insert the following paragraph immediately after line 6 and before line 7:

FIG. 15. Mesoscopic 4-terminal SQUID qubit. The  $\pi$ -phase shifter in the flux loop is added to attain bistability without external flux. The externally controlled transport current  $I$  affects the current  $J$  in the superconducting loop through the phase dragging effect and in turns in the loop.

On page 11, please insert the following paragraph immediately after line 6 and before line 7:

FIG. 16. Two coupled five-terminal qubits. The left qubit has an asymmetric current configuration while the right one has a symmetric one.

On page 11, please insert the following paragraph immediately after line 6 and before line 7:

FIG. 17. System of  $N$  coupled qubits. The distance between the flux regions is maximized to have less magnetic interaction between the qubits

On page 11, line 19, please replace the paragraph beginning "Intrinsic phase shifters" with the following paragraph:

Intrinsic phase shifters in superconducting phase quantum bits (qubits) are disclosed in M. H. S. Amin, T. Duty, A. Omelyanchouk, G. Rose and A. Zagoskin, U.S. Provisional Application Serial No. 60/257624, "Intrinsic Phase Shifter as an Element of a Superconducting Phase Quantum Bit", filed Dec. 22, 2000, herein

incorporated by reference in its entirety. A phase shifting structure with 0 and  $\pi$ -phase shifts in a two-terminal DC SQUID is described in R. R. Schulz, B. Chesca, B. Goetz, C. W. Schneider, A. Schmehl, H. Bielefeldt, H. Hilgenkamp, J. Mannhart and C. C. Tsuei, "Design and Realization of an all d-Wave dc  $\pi$ -Superconducting Quantum Interference Device", Appl. Phys. Lett. 76, 7 ~~p.912-14~~ p. 912-14 (2000), herein incorporated by reference in its entirety.

On page 13, line 16, please replace the paragraph beginning "The two degenerate states" with the following paragraph:

The two degenerate states, corresponding to classical bit states, represented symbolically as  $|0\rangle$  and  $|1\rangle$ , are then the two basis states of the qubit quantum system of qubit 100. The magnitude of the flux threading superconducting loop 122 can be much less than half a flux quantum  $\Phi_0$ , both because of intrinsic phase shifter 123 and the presence of the terminals 110-1 through 110-N, which also introduces a phase shift. At least two external terminals, terminals 110-2 and 110-j in Figure 42 ~~13~~, for example, of qubit 100 can be coupled to sources of transport current  $I_i$  and  $I_j$ , respectively, in Figure 42 ~~13~~. Terminals 110-i and 110-j along with the current source creates a transport current loop 127. Additionally, an external magnetic flux, indicated by field  $\vec{B}$ , can be applied through superconducting loop 122 in order to control the physical parameters of the qubit quantum system of qubit 100. By changing the transport current  $I_T$  and/or applying an external magnetic field  $\vec{B}$ , the magnitude of the flux threading superconducting loop 122, the potential barrier between the two basis states  $|0\rangle$  and  $|1\rangle$  of the qubit quantum system of qubit 100, and the tunneling matrix element  $\Delta_T(I)$  between the basis states of the qubit quantum system can be adjusted.

On page 18, please replace the paragraph beginning "In an exemplary embodiment" (line 24) with the following paragraph:

In an exemplary embodiment, four-terminal constriction junction 120 can be fabricated of aluminum. Widths  $W_1$  and  ~~$W_{2A}$~~   $W_2$  can each be approximately 0.5 microns; widths  $W_3$  and  $W_4$  can each be approximately 0.05 microns; lengths  $L_1$ ,  $L_4$ ,

L<sub>5</sub>, L<sub>6</sub>, L<sub>7</sub>, and L<sub>8</sub> can each be approximately 1 micron; and lengths L<sub>2</sub> and L<sub>3</sub> can each be approximately 0.55 microns.

On page 19, line 30, please replace the paragraph beginning with "Tunnel junction 140 can be fabricated" with the following paragraph:

Tunnel junction 140 can be fabricated where insulation layer 150 (FIG. 1C) is an aluminum oxide layer of thickness approximately 0.05 microns. Insulating layer 150 can be deposited onto terminal 110-4. Terminal 110-1 is then deposited on insulating layer 150. The tunnel junction 141 can also be fabricated where insulating layer 150 (FIG. 1C) is an aluminum oxide layer of thickness approximately 0.05 microns which is deposited onto terminal 110-3, upon which is deposited terminal 110-2. Although in FIG. 1C, terminals 110-1 and 110-2 are shown above terminals 110-4 and 110-3, respectively, in general terminals 110-1 and 110-2 can be on either side of terminals 110-4 and 110-3, respectively.

On page 56 please replace the paragraph beginning with "In the past twenty years" with the following paragraph:

In the past twenty years, clear evidences have been gained that quantum information processing could ~~offers~~ offer significant advantages over classical information processing [1, 2]. In parallel, it was recognized that superconducting systems, and particularly SQUIDs, are good candidates for the observations of quantum ~~phenomenas~~ phenomena at the macroscopic level [3]. A great body of experimental evidences has been accumulated to support this assertion.

On page 56 please replace the paragraph beginning with "In this paper" with the following paragraph:

In this paper, a novel qubit design using the phase degree of freedom is ~~introduce~~ introduced. Similarly to the rf-SQUID [4] or to the design introduced in [5] the logical states are represented by persistent currents of different orientations flowing in a superconducting loop. In the present design logical operations are not

performed by manipulations of external fluxes. This is, among others, an advantage of this design.

On page 56 please replace the paragraph beginning with “The basic constituent” with the following paragraph:

The basic constituent of the novel qubit design studied in this paper is the 4-terminal Josephson junction (~~FIG. 1~~ Fig. 14) which has been the focus of extensive studies [7]. Nonlocal coherent transport in this structure was investigated in the companion paper (thereon referred to as Article I). The ~~mesoscopic~~ mesoscopic 4-terminal junctions of ~~FIG. 1b) and c)~~ 14(b) and 14(c) on which we will focus our attention in this paper consist of four s-wave superconducting banks weakly coupled by a ballistic two-dimensional electron gas (2DEG) layer. As seen in Article I, in such devices the total current,  $I_i$ , flowing into the  $i$ -th terminal depends on the superconducting phases  $\varphi_j$  at all the banks and has the form[8]:

$$I_i = \frac{\pi\Delta_o}{e} \sum_{j=1}^4 \gamma_{ij} \sin\left(\frac{\varphi_i - \varphi_j}{2}\right) \tan h\left[\frac{\Delta_o}{2T} \cos\left(\frac{\varphi_i - \varphi_j}{2}\right)\right],$$

where  $\Delta_0$  is the superconducting gap in the banks, the  $\gamma_{ij}$  are geometry dependent coupling coefficients between the  $i^{\text{th}}$  and  $j^{\text{th}}$  terminals and  $\gamma_{ij}=0$ . ~~A Pair pair~~ of terminals are either current biased or closed by a superconducting loop (see below). As noted ~~on in~~ FIG. ~~1~~ 14, we label the terminals such that terminals 1 and 2 sustain current  $I$  while terminal 3 and 4 current  $J$ ; below  $I$  and  $J$  will respectively be transport and persistent currents.

Please delete the figure at the top of page 57.

On page 57 please delete the paragraph beginning with “Fig. 1” as follows:

~~FIG. 1. 4 terminal Josephson junctions. (a) Junction with microbridges. (b) Mesoscopic junction with two dimensional electron gas (2DEG) and symmetric current configuration. In this configuration, coupling between currents  $I$  and  $J$  inside~~

~~the normal region is local ( $\delta=0$ ). (c) Mesoscopic junction in the asymmetric currents configuration. Coupling is non-local ( $\delta\neq 0$ ).~~

On page 57 please replace the paragraph beginning with “For multi-terminal junctions” with the following paragraph:

For multi-terminal junctions such that  $\det(\gamma_{\text{coup}})=0$ , coupling between the currents I and J is local. Constriction junctions (FIG. 1a 14a) and mesoscopic junctions in the crossed currents configuration (FIG. 2b 14b) ~~belongs~~ belong to this category. In the later case, although coupling is really nonlocal, it is the extra symmetry in the current configuration that imposes  $\det(\gamma_{\text{coup}})=0$ . Alternatively, when  $\det(\gamma_{\text{coup}})\neq 0$  coupling is nonlocal. This is realized for the mesoscopic junction in the parallel current configuration (FIG. 1e 14c). In this case, and as emphasized in Article I, the phase across the two terminals in one current loop is affected by the direction of the current in the other loop. This will be referred to as phase dragging effect. As will be shown below, this effect plays a crucial role in manipulation, initialization and readout of the qubit's states.

On page 57 please replace the paragraph beginning with “By connecting terminals 3 and 4” with the following paragraph:

By connecting terminals 3 and 4 via a superconducting ring, a 4-terminal SQUID is realized. FIG. 2 15 (the  $\pi$ -junction will be introduced below) shows such a structure where, to take advantage of phase dragging effect, the asymmetric ~~currents~~ current configuration of FIG. 1e 14c is used.

On page 57 please replace the paragraph beginning with “In this configuration” with the following paragraph:

In this configuration, the 4-terminal SQUID is reminiscent of the much studied rf-SQUID [9]. It is ~~know~~ known that for rings of large enough inductance and under  $\Phi_0/2$  externally applied flux ( $\Phi_0=h/2e$ , the flux quantum) the rf-SQUID can be bistable [4]. The bistable states correspond to different orientations of persistent

currents or equivalently to different magnetic flux enclosed by the ring. Quantum behavior of the flux degree of freedom in this system as has been the focus of numerous experimental and theoretical investigations over the past twenty years, see [4] for reviews.

On page 57 please replace the paragraph beginning with “Bistability of the 4-terminal” with the following paragraph:

Bistability of the 4-terminal SQUID ~~as~~ has also been the focus of theoretical investigations and ~~as~~ has been confirmed experimentally [10]. For quantum computing purposes, a major advantage of the 4-terminal SQUID over the standard rf-SQUID is that bistability can be attained for arbitrarily small loop inductance. As a result, the enclosed flux is much smaller than for the rf-SQUID and weaker coupling to the environment is to be expected. Small inductances and thus small fluxes ~~where~~ were the main advantage of the 3 ~~junctions~~ junction design of reference [5].

On page 57 please replace the paragraph beginning with “As in the rf-SQUID” with the following paragraph:

As in the rf-SQUID or 3 ~~junctions~~ junction case, an external flux of  $\Phi_e = \Phi_0/2$  is needed to get bistability (see appendix A). However, as will become clear in the next sections, this control parameter is not exploited to manipulate the flux (i.e. qubit) state. The external flux can thus be fixed to  $\Phi_0/2$  for all qubits. This opens the possibility of replacing, as shown in FIG. 2 15, the external fluxes by a  $\pi$ -phase shifter in each qubit's superconducting ring. The net effect is the same but this has the advantage that the  $\pi$ -phase shifter does not bring in extra coupling to the electromagnetic environment.

Please delete the figure at the top of page 58.

On page 58 please delete the paragraph beginning with “Fig. 2” as follows:

~~FIG. 2. Mesoscopic 4-terminal SQUID qubit. The  $\pi$  phase shifter in the flux loop is added to attain bistability without external flux. The externally controlled transport current  $I$  affects the current  $J$  in the superconducting loop through the phase dragging effect and in turns the flux in the loop.~~

On page 58 please replace the paragraph beginning with “In mesoscopic four terminal junctions where” with the following paragraph:

In mesoscopic four terminal junctions where the 2DEG layer as has a square geometry, the coefficient  $\gamma_{ij}$  are such that  $\gamma_{13} = \gamma_{24}$  and  $\gamma_{14} = \gamma_{23}$ . As shown in the appendix, this constrains  $\chi$  to take only one of two possible values, 0 and  $\pi$ . We will thereon work under this condition but results can be generalized to the more general case. We define the new variables  $\gamma = (\gamma_{13} + \gamma_{14}) / \gamma_{12}$  and  $\delta = (\gamma_{13} - \gamma_{14}) / \gamma_{12}$ . The difference between local and nonlocal couplings lies in the value of  $\delta$ , namely  $\delta = 0$  for the local case but  $\delta \neq 0$  in the nonlocal case.

On page 58 please replace the paragraph beginning with “In the non-locally coupled case” with the following paragraph:

In the non-locally coupled case, the bias term can be used to ~~breaks~~ break this degeneracy and make one of the minima more favorable than the other. Hence, bias energy and barrier height can be tuned by the externally applied transport current. In the following sections, we will show how these properties can be used for initialization, readout and manipulation of the qubit's state.

On page 60 please replace the paragraph beginning with “In the asymmetric current configuration” with the following paragraph:

In the asymmetric current configuration of FIG. ~~2~~ 15,  $\delta \neq 0$  and the transport current will also produce an energy bias. When the bias exceeds the level width, tunneling between the two basis states stops because they are out of degeneracy [11]. Hence, if  $I=0$ , the levels are degenerate but the qubit is frozen because the barrier height is at it's maximum. When  $I=1$  however,  $\epsilon(I)$  reaches it's maximum value  $\sqrt{2} \delta$



and the barrier height it's minimum value. Coherent tunneling is nevertheless prohibited because the degeneracy condition is not satisfied. As a result, for a  $\delta \neq 0$  a qubit can either be frozen ( $I=0$ ) or the effective Hamiltonian  $\varepsilon(I)\sigma_z$  is generating a phase difference between the basis states ( $I>0$ )

$$Z(\beta) = e^{-i\sigma_z\beta/2},$$

with  $\beta = 2E_J\varepsilon(I)t/\hbar$ .

On page 61 please replace the paragraph beginning with "It is well known" with the following paragraph:

It is well known that to implement arbitrary single qubit logical operations, one needs rotations around at least two orthogonal axis [12]. Since the situations  $\delta=0$  and  $\neq 0$  cannot be realized in the same 4-terminal SQUID we clearly have not yet ~~have~~ reached this goal. Different ways to remedy to this are possible. First, for  $\delta \neq 0$  the 4-terminal SQUIDS can be engineered such that for  $I=0$  tunneling is not suppressed (e.g. by working with smaller capacitances). In this case, both  $X(\alpha)$  and  $Z(\beta)$  operations are possible. It was shown in [13] how, in that situation, to 'freeze' a qubit evolution using NMR-like refocusing sequences. Alternatively, for  $\delta=0$ , it will become clear in section VII that by using two-qubit gates and the single-qubit gate  $X(\alpha)$ , one can implement arbitrary single-qubit gates. Finally, we will see in the next section how the 4-terminal SQUID can be modified to effectively combine both the  $\delta=0$  and  $\delta \neq 0$  cases.

On page 61 please replace the paragraph beginning with "As exposed in the previous section" with the following paragraph:

As exposed in the previous section, junctions with  $\delta=0$  and  $\delta \neq 0$  offer complementary possibilities for single bit operations and it would be practical to combine those possibilities in a single qubit design (this would for example remove the need of refocusing to freeze qubits in the asymmetric current configuration). This is impossible for the single 4-terminal SQUID because, as seen in FIG. ~~4b)~~ 14b) and

e) 14c), non-local and local coupling of currents inside the 2DEG layer are realized in differently engineered junctions.

On page 61 please replace the paragraph beginning with “We suggest here to use” with the following paragraph:

We suggest here to use the 5-terminal junctions illustrated in FIG. 3 16. Coupling between the two flux rings is used to create entanglement and will be discussed in the next section. The extra lead for transport current in this design gives the opportunity to use either symmetric ( $\delta=0$ ) or asymmetric ( $\delta\neq 0$ ) current configurations. More specifically, the qubit on the left hand side of FIG. 3 16 (where the middle junction is disconnected from the current source) is completely equivalent to the asymmetric 4-terminal SQUID. In this configuration, the transport current tunes  $\varepsilon(I)$  (as discussed in the previous section, it will also tune  $\delta(I)$  but as long as the truncation conditions hold, this does not change the dynamics). On the other hand, the rightmost qubit in FIG. 3 16 is in the symmetric configuration and therefore  $\delta = 0$ . For the current values indicated on the leads of this qubit, the description is analogue analogous to the 4-terminal case already discussed. Therefore, in this symmetric configuration, the transport current tunes  $\Delta(I)$  while  $\varepsilon(I)$  is fixed to zero.

On page 62, please delete Fig. 3.

On page 62, please delete the paragraph beginning “Fig. 3” as follows:

~~Fig. 3. Two coupled five-terminal qubits. The left qubit has asymmetric current configuration while the right one has symmetric one.~~

On page 63, please delete Fig. 4.

On page 63, please delete the paragraph beginning “Fig. 4” as follows:

~~Fig. 4. System of  $N$  coupled qubits. The distance between the flux regions is maximized to have less magnetic interaction between the qubits.~~

On page 62, please delete the paragraph beginning “4-terminal case already discussed” as follows:

~~4-terminal case already discussed. Therefore, in this symmetric configuration, the transport current tunes  $\delta(I)$  while  $\epsilon(I)$  is fixed to zero.~~

On page 62 please replace the paragraph beginning with “To obtain a universal set of gates” with the following paragraph:

To obtain a universal set of gates for the 5-terminal qubits, a prescription for a non-trivial qubit-qubit interaction is needed [12]. Here again, this interaction is provided by the phase dragging effect. Two qubits are coupled through an additional mesoscopic 4-terminal junctions as presented schematically in FIG. 3 16.

On page 63 please replace the paragraph beginning with “As schematically shown in figure” with the following paragraph:

As schematically shown in FIG. 4 17, N qubits can be connected in this way to form an N-qubit quantum register. From the 2-qubit logical operation  $CP(\zeta)$  and the single-qubit gates defined earlier, it is possible to implement arbitrary N-qubit logical operations on such a register [13]. For example, a Controlled-NOT gate can be implemented, up to an irrelevant global phase factor, by the following sequence of one- and two-qubit gates

$$CN_{12}=X_2(3\pi/2)CP_{12}(\pi/2)Z_2(\pi/2)X_2(\pi/2)CP_{12}(\pi/2)Z_2(\pi/2)Z_1(\pi/2). \quad (24)$$

On page 63, please replace the paragraph beginning with “Moreover, figure 4 shows an” with the following paragraph:

Moreover, FIG. 17 shows an N-qubit array in its simplest design: a linear array of qubits limited to nearest-neighbor couplings. While more complex coupling schemes are possible, this configuration is, at least for some computational tasks, sufficient [16]. Finally, as announced in section V, for two-coupled 4-terminal

SQUIDS in the symmetric configuration (i.e. where  $\delta=0$  for both qubits), the two-qubit gate  $CP(\zeta)$  and the single-qubit gate  $X(\alpha)$  are universal. This is quite obvious because  $CP(\zeta)$  gives the possibility of accumulating relative phases, something which was absent when only  $X(\alpha)$  was available.

On page 64, please replace the paragraph beginning with “For typical junction parameters” with the following paragraph:

For typical junction parameters, the Josephson energy  $E_0 = \hbar I_0 / 2e$  is of the order of XX. For the corresponding frequency of operation of the qubit, the lead impedance is of the order of the vacuum impedance and thus  $R_I \sim 100\Omega$  [4, 22]. Since  $\delta \ll 1$ , for working temperatures in the ~~milliKelvin~~ milliKelvin range, the quality factor should be large. The ratio  $R_I / R_K \sim 10^{-3}$  is the limiting factor here. However, as we shall shortly see, this ratio could potentially be modulated.

On page 64, please replace the paragraph beginning with “Moreover, care must be taken” with the following paragraph:

Moreover, care must be taken that the ac Josephson voltage generated by the phase fluctuations does not generate inter-Andreev level transitions [19]. This imposes  $\omega_0 < E_s$  which therefore requires that  $C > 1 / LE_g^2$ . This ~~constrain~~ constraint on the capacitance does not contradict the condition (20) already imposed in section V to minimize bit flip errors.

On page 64, please replace the paragraph beginning with “Pair of qubits are also coupled” with the following paragraph:

Pairs of qubits are also coupled to the electromagnetic environment through the gate voltage used to tune qubit-qubit interactions. When this interaction is turned off, i.e. when the coupling 4-terminal junction of FIG. 3 16 is depleted, small fluctuations of the voltage should have no effect on qubits as long as the gate voltage is large enough. This is because the fluctuations of voltage introduce fluctuations in phase  $\varphi'$  (through the parameter  $k'$ , see appendix B). Since  $\varphi' \ll \varphi$  ( $k' \gg \gamma$ ), these

fluctuation couple very weakly to the state of qubits and therefore do not introduce considerable decoherence. Moreover, these fluctuations can be minimized by working with a low resistance voltage source [4, 20]. When the voltage is turned off, the region is no longer depleted and the qubit-qubit interaction is on. In this case, since the gate voltage is zero, the voltage source can be practically disconnected from the circuit and therefore no decoherence will be introduced.